A discussion of the Symmetric Bayesian Nash Equilibrium in Kyle 1989

Preliminary Draft

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Abstract

This paper revisits the Symmetric Bayesian Nash Equilibrium in *Informed speculation with imperfect competition* and tries to argue that the original solution is not a Nash Equilibrium. This mistake arises from imposing symmetry of strategy before calculating the best response function. To correct this mistake, this paper presents an appropriate solution to the symmetric Bayesian Nash equilibrium. In addition, I will show through simulation that my solution is qualitatively different from Kyle's equilibrium.

1 Introduction

Kyle (1989) is an important paper on market micro structure. It solves the "schizophrenia" problem of the competitive rational expectation equilibrium.[1] This paper models asset market as a strategic game where each agent submits a demand schedule. An auctioneer clears the market with an equilibrium price $\tilde{p}$ such that aggregate supply equals aggregate demand.

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[1] The "schizophrenia" problem of REE is introduced by Hellwig (1980). It refers to the problem that traders take equilibrium price as given while their trading positions affect the equilibrium prices.

Here's a brief review of the setup of this model. There is a single risky asset $\tilde{v}$ whose return follows a normal distribution $\tilde{v} \sim \mathcal{N}(0, \sigma_v^2)$. The market consists informed traders indexed as $n \in \{1, 2, ...N\}$, uninformed traders indexed as $m \in \{1, 2..., M\}$ and noise traders whose demand $z \sim \mathcal{N}(0, \sigma_z^2)$.

Informed traders receives a private information $i_n = \tilde{v} + e_n$ where e_n is assumed to be independent of $\tilde{v}$ and its distribution is $\mathcal{N}(0, \sigma_e^2)$. All traders are assumed to have identical CARA utility function with risk aversion coefficient equals ρ_n for the informed and ρ_m for the uninformed.

However, noise traders are not assumed to maximize any utility function. Joint normality is a critical assumption in this model. The vector $(\tilde{v}, i_1, i_2, ...i_N, z)$ is distributed with mean 0 and some variance covariance matrix Σ.

This paper considers the symmetric Bayesian Nash equilibrium as the solution concept. Traders are assumed to have market power and the Bayesian Nash equilibrium takes the price effect into account. The price impact of traders are denoted as λ_n for the informed traders and λ_m for the uninformed.

The strategies (demand schedule) $X_n(i_n, p)$ and $Y_m(p)$ are Bayesian Nash equilibrium if and only if:

$$\mathbb{E}\{u_n(\tilde{v} - p(X,Y))X_n(X,Y)\} \geq \mathbb{E}\{u_n(\tilde{v} - p(X',Y))X_n'(X',Y)\} \tag{1}$$

$$\mathbb{E}\{u_m(\tilde{v} - p(X,Y))Y_m(X,Y)\} \geq \mathbb{E}\{u_m(\tilde{v} - p(X,Y'))Y_m'(X,Y')\} \tag{2}$$

where X_n' and Y_m' are any deviating strategies. An important note is the strategy profile X (Y) and X' (Y') only differ at the nth (mth) component.

Given the CARA and normal random variable setup, this paper shows the optimal strategy is characterized by the following:

$$X_n(i_n, p) = \frac{\mathbb{E}(\tilde{v} - p|i_n, p)}{\lambda_n + \rho_n Var(\tilde{v} - p|i_n p)} \tag{3}$$

$$Y_m(p) = \frac{\mathbb{E}(\tilde{v} - p|p)}{\lambda_m + \rho_m Var(\tilde{v} - p|p)} \tag{4}$$

λ_n and λ_m in the formula represents the price impact, which will be determined endogenously. In addition, ρ_n and ρ_m represents the risk aversion coefficient for informed and uninformed trader respectively .

2 Kyle's solution

In this section, I will explain the original price formation in Kyle (1989) and show that the way it imposes symmetry violates the Nash equilibrium. Let me first discuss Kyle's solution.

In this model, traders are assumed to uncover the aggregate information from asset price. In equilibrium, price will clear the market:

$$\sum_{n=1}^{N} X_n(i_n, p) + \sum_{m=1}^{M} Y_m(p) + z = 0$$

Consider a linear equilibrium. Let $X_n = \mu_{I,n} + \beta_n i_n - \gamma_{I,n} p$ denote the strategy of informed trader n and $Y_m = \mu_{U,m} - \gamma_{U,m} p$ denote the strategy of uninformed trader m. Market clears if we have:

$$\sum_{n=1}^{N} (\mu_{I,n} + \beta_n i_n - \gamma_{I,n} p) + \sum_{m=1}^{M} (\mu_{U,m} - \gamma_{U,m} p) + z = 0$$

Kyle's paper considers a symmetric linear equilibrium with $X_n = \mu_I + \beta i_n - \gamma_I p$ and $Y_m = \mu_U - \gamma_U p$. At the symmetric Bayesian Nash equilibrium, price clears the market with the following equation:

$$(N\mu_I + N\beta\tilde{v} + \beta \sum_{n=1}^{N} e_n - N\gamma_I p) + M(\mu_U - \gamma_U p) + z = 0 \tag{5}$$

Solving for price, we get

$$\tilde{p} = (N\gamma_I + M\gamma_U)^{-1}[N\beta\tilde{v} + \beta(\sum_{n=1}^{N} e_n) + z + N\mu_I + M\mu_U]$$

For the uninformed trader m, price $\tilde{p}$ is informationally equivalent as $\tilde{h}$:

$$\tilde{h} = (N\beta)^{-1}[(N\gamma_I + M\gamma_U)\tilde{p} - N\mu_I - M\mu_U + z] \tag{6}$$

$$= \tilde{v} + \frac{\sum_{n=1}^{N} e_n}{N} + \frac{z}{N\beta} \tag{7}$$

For the informed trader, since his strategy is measurable with respect to i_n and p, we can separate i_n from equation (5). Price is informationally equivalent as $\tilde{h}_n$ with:

$$\tilde{h}_n = [(N-1)\beta]^{-1}[(N\gamma_I + M\gamma_U)\tilde{p} - \beta i_n - N\mu_I - M\mu_U + z] \quad (8)$$
$$= \tilde{v} + \frac{\sum_{k\neq n} e_k}{N-1} + \frac{z}{(N-1)\beta} \quad (9)$$

Equation (7) and (9) suggest we have the following conditional expectations and conditional variances from the normal projection theorem if we impose the symmetry from equation (5).

Conditional variances:

$$Var(\tilde{v}|\tilde{p}) = Var(\tilde{v}|\tilde{h}) = [\frac{1}{\sigma_v^2} + \frac{N^2\beta^2}{N\beta^2\sigma_e^2 + \sigma_z^2}]^{-1} \quad (10)$$
$$Var(\tilde{v}|i_n, \tilde{p}) = Var(\tilde{v}|i_n, \tilde{h}_n) = [\frac{1}{\sigma_v^2} + \frac{1}{\sigma_e^2} + \frac{(N-1)^2\beta^2}{(N-1)\beta^2\sigma_e^2 + \sigma_z^2}] \quad (11)$$

Conditional expectations:

$$\mathbb{E}(\tilde{v}|\tilde{p}) = \mathbb{E}(\tilde{v}|\tilde{h}) = \frac{Var(\tilde{v}|\tilde{h})}{Var(\bar{e} + \frac{z}{N\beta})}\tilde{h} \quad (12)$$
$$\mathbb{E}(\tilde{v}|i_n, \tilde{p}) = \mathbb{E}(\tilde{v}|i_n, \tilde{h}) = \frac{Var(\tilde{v}|i_n, \tilde{h}_n)}{\sigma_e^2} i_n + \frac{Var(\tilde{v}|i_n, \tilde{h}_n)}{Var(\bar{e}_{-n} + \frac{z}{(N-1)\beta})}\tilde{h}_n \quad (13)$$

Plug in the expression of the conditional expectations and conditional variances into FOC (3) (4) and substitute the expression of $\tilde{h}$ and $\tilde{h}_n$ yields the solution in Kyle (1989).

3 A discussion on the SBNE

To distinguish from Kyle's solution, I denote the strategies of the informed and the uninformed respectively as:

$$X_n = \mu_n + \beta_n i_n - \gamma_n p$$

$$Y_m = \psi_m - \delta_m p$$

The definition of symmetric Bayesian Nash equilibrium is the following:

Definition 1. *Bayesian Nash equilibrium*

A strategy profile for the informed traders $(\boldsymbol{\mu}, \boldsymbol{\beta}, \boldsymbol{\gamma})$ *is a Bayesian Nash equilibrium if fixing others' strategies, for any* $n \in \{1, 2, ...N\}$ *such that* $(\mu_n, \beta_n, \gamma_n)$ *is a best response to the strategy* *profile* $(X_1, X_2, ...X_{n-1}, X_{n+1}, ...X_N; Y_1, ...Y_M)$. *Similarly, the Bayesian Nash equilibrium for uninformed traders is defined as for any* $m \in \{1, 2, ...M\}$, (ψ_m, δ_m) *is the best response to the strategy profile* $(X_1, X_2, ..., X_N; Y_1, ...Y_{m-1}, Y_{m+1}, ..., Y_M)$.

Definition 2. *Symmetric Bayesian Nash equilibrium*

A SBNE is a Nash equilibrium such that $X_1 = X_2 = ... = X_N$ *and* $Y_1 = Y_2 = ... = Y_M$

Given the definition of SBNE, we know that the solution must be a fixed point of the best response functions. However, the symmetry condition should not be imposed while solving the best response function. In this model, price plays 2 roles. In equilibrium, price clears the market. However, price also transmits aggregate information to the market.

At the symmetric Bayesian Nash equilibrium, price clears market with the following equation.

$$(N\mu + N\beta\bar{i} - N\gamma p) + M(\psi - \delta p) + z = 0$$

However, in terms of information transmission for the informed traders, price formation should be written in the following way:

$$p = \frac{X_n}{(N-1)\gamma + M\delta} + \frac{(N-1)\beta\bar{i}_{-n}}{(N-1)\gamma + M\delta} + \frac{z}{(N-1)\gamma + M\delta} + \frac{(N-1)\mu + M\psi}{(N-1)\gamma + M\delta} \tag{14}$$

where X_n is the choice variable of individual n. The market impact is derived by

$$\lambda_n = \frac{\partial p}{\partial X_n} = \frac{1}{(N-1)\gamma + M\delta}$$

Since demand schedule $X_n(i_n, p)$ is a measurable function with respect to (i_n, p), price is informationally equivalent as the residual supply function h_n:

$$\begin{aligned} h_n &= \frac{(N-1)\gamma + M\delta}{(N-1)\beta}[p - \frac{X_n}{(N-1)\gamma + M\delta} - \frac{(N-1)\mu + M\psi}{(N-1)\gamma + M\delta}] \\ &= \tilde{v} + \bar{e}_{-n} + \frac{z}{(N-1)\beta} \end{aligned}$$

Kyle's mistake arises from the imposing $X_n = \mu + \beta i_n - \gamma p$ in equation (14). Therefore the price formation is equivalent as the following:

$$p = \frac{N\beta\bar{i} + N\mu + M\psi + z}{N\gamma + M\delta} \tag{15}$$

$$p = \frac{\mu + \beta i_n + (N-1)\beta\bar{i}_{-n} + (N-1)\mu + M\psi + z}{\gamma + (N-1)\gamma + M\delta} \tag{16}$$

However, let's consider a marginal deviation of player n's strategy. The price impact of this deviation is given by:

$$\frac{\partial p}{\partial \beta} = \lambda i_n + \lambda(\sum_{j \neq n}^{N} i_j) \neq \frac{\partial p}{\partial X_n}\frac{\partial X_n}{\partial \beta} = \lambda_n i_n$$

with $\lambda = (N\gamma + M\delta)^{-1}$. The actual price impact for a marginal deviation for trader n is given at the right hand side. However on the left hand side, it seems that this deviation has a stronger impact. This problem occurs because the previous analysis is constrained at the symmetric equilibrium, which leave no room for a marginal deviation for one player without affecting other players.

Moreover, if we compare the residual supply function of $\tilde{h}_n$ in Kyle's solution with h_n, we find $\tilde{h}_n$ not necessarily equals h_n. In addition, $\tilde{h}_n$ equals h_n if and only if $(\mu_n, \beta_n, \gamma_n)$ are equal to (μ, β, γ), which suggests the residual supply function should have the same relative slope, i.e. β and γ, as the strategy of player n, i.e. β_n and γ_n. However, Nash equilibrium requires X_n is a best response given the strategy of other trader. A symmetric Nash equilibrium is a situation where (β, γ) is a best response when other traders play (β, γ). According to Kyle's construction, equation

$$\tilde{h}_n = \frac{\sum_{k \neq n} e_k}{N-1} + \frac{z}{(N-1)\beta}$$

implies a trader could improve the information precision by simply choosing a larger value of β. However, this yields a paradox because $\tilde{h}_n$ represents the residual supply which should not be affected by any particular trader.

The following equation characterizes the price formation for the uninformed.

$$p = \frac{Y_m}{(M-1)\delta + N\gamma} + \frac{N\beta\bar{i}}{(M-1)\delta + N\gamma} + \frac{z}{(M-1)\delta + N\gamma} + \frac{N\mu + (M-1)\psi}{(M-1)\delta + N\gamma} \tag{17}$$

The price impact is derived as

$$\lambda_m = \frac{1}{(M-1)\delta + N\gamma}$$

Price is informationally equivalent as the residual supply:

$$\begin{aligned} h_m &= \frac{1}{\lambda_m N\beta}[p - \lambda_m Y_m - \frac{(M-1)\psi + N\mu}{(M-1)\delta + N\gamma}] \\ &= \tilde{v} + \bar{e} + \frac{z}{N\beta} \end{aligned}$$

4 Proposed solution

In this section, I will show the appropriate method to solve the SBNE in this game. It is a modification of Kyle's solution without imposing symmetry at the price formation stage. Here's the symmetry assumption I impose:

Assumption 4.1. *Imposing symmetry*
For any informed trader $n \in \{1,2,3,...N\}$, he believes the following

$$\beta_1 = \beta_2 = ... = \beta_{n-1} = \beta_{n+1} = ...\beta_N = \beta$$

$$\gamma_1 = \gamma_2 = ... = \gamma_{n-1} = \gamma_{n+1} = ...\gamma_N = \gamma$$

$$\mu_1 = \mu_2 = ... = \mu_{n-1} = \mu_{n+1} = ...\mu_N = \mu$$

and

$$Y_1 = Y_2 = ...Y_M = \psi - \delta p$$

Similarly, for any uninformed trader $m \in \{1,2,...,M\}$, she believes the following:

$$\psi_1 = \psi_2 = ... = \psi_{m-1} = \psi_{m+1} = ...\psi_M = \psi$$

$$\delta_1 = \delta_2 = ... = \delta_{m-1} = \delta_{m+1} = ...\delta_M = \delta$$

and

$$X_1 = X_2 = ... = X_N = \mu + \beta i_n - \gamma p$$

This assumption suggests each trader takes others' strategies as given and they are symmetric. In addition, this assumption also states that informed and uninformed traders hold the same belief on the value of $(\mu, \beta, \gamma; \psi, \delta)$, which is another convenient assumption.

I will introduce some notations to simplify the analysis. Since for informed traders, price is informationally equivalent as the random intercept $h_n = \tilde{v} + \bar{e}_{-n} + \frac{z}{(N-1)\beta}$, let the random variable

$$s = \bar{e}_{-n} + \frac{z}{(N-1)\beta}$$

And s has a normal distribution with mean 0 and variance

$$\sigma_s^2 = \frac{\sigma_e^2}{N-1} + \frac{\sigma_z^2}{((N-1)\beta)^2}$$

Similarly for the uninformed traders, let the random variable

$$r = \bar{e} + \frac{z}{N\beta}$$

While r is distributed normally with mean 0 and variance

$$\sigma_r^2 = \frac{\sigma_e^2}{N} + \frac{\sigma_z^2}{N^2\beta^2}$$

Lemma 4.2. *The conditional variance $Var(\tilde{v}|i_n, p) = \sigma_c^2$ for informed traders is calculated by*

$$\frac{1}{\sigma_c^2} = \frac{1}{\sigma_v^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_s^2}$$

Proof. We have $\sigma_c^2 = Var(\tilde{v}|i_n, p) = Var(\tilde{v}|\tilde{v} + e_n, \tilde{v} + s)$. Hence the projection theorem of normal distribution implies :

$$\frac{1}{\sigma_c^2} = \frac{1}{\sigma_v^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_s^2}$$

since we have s being independent of e_n. □

Lemma 4.3. *The conditional expected value for the informed*

$$\mathbb{E}\{\tilde{v}|i_n, p\} = \mathbb{E}\{\tilde{v}|i_n, h_n\} = \frac{\sigma_c^2}{\sigma_e^2} i_n + \frac{\sigma_c^2}{\sigma_s^2} h_n$$

Proof. Since p is informationally equivalent as the random intercept h_n, the first equality holds. The second equality is just an application of the projection theorem for normal distribution. □

As an analogous, the conditional variance and expected value for the uninformed is given by:

Lemma 4.4. *The conditional variance for the uninformed $Var(\tilde{v}|p) = \sigma_m^2$ is calculated by*

$$\frac{1}{\sigma_m^2} = \frac{1}{\sigma_v^2} + \frac{1}{\sigma_r^2}$$

and the conditional expected value is given by

$$\mathbb{E}\{\tilde{v}|p\} = \mathbb{E}\{\tilde{v}|h_m\} = \frac{\sigma_m^2}{\sigma_r^2} h_m$$

Proof. As for uninformed traders, we have $\mathbb{E}(\tilde{v}|p) = \mathbb{E}(\tilde{v}|h_m) = \mathbb{E}(\tilde{v}|\tilde{v} + r)$. Therefore the proof is identical to the proof of previous lemma. It is another application of the projection theorem for the normal distribution. □

Theorem 4.5. *The best response function is characterized as the following equation systems.*

$$\beta^* = \frac{\sigma_c^2}{\sigma_e^2}[\lambda_n + \rho_n\sigma_c^2 + \frac{\sigma_c^2}{\sigma_s^2}\frac{1}{(N-1)\beta}]^{-1} \tag{18}$$

$$\gamma^* = [1 - \frac{\sigma_c^2}{\sigma_s^2}\frac{1}{\lambda_n(N-1)\beta}][\lambda_n + \rho_n\sigma_c^2 + \frac{\sigma_c^2}{\sigma_s^2}\frac{1}{(N-1)\beta}]^{-1} \tag{19}$$

$$\mu^* = -\frac{\sigma_c^2}{\sigma_s^2}\frac{(N-1)\mu + M\psi}{(N-1)\beta}[\lambda_n + \rho_n\sigma_c^2 + \frac{\sigma_c^2}{\sigma_s^2}\frac{1}{(N-1)\beta}]^{-1} \tag{20}$$

$$\delta^* = [1 - \frac{\sigma_m^2}{\sigma_r^2}\frac{1}{\lambda_m N\beta}][\lambda_m + \rho_m\sigma_m^2 + \frac{\sigma_m^2}{\sigma_r^2}\frac{1}{N\beta}]^{-1} \tag{21}$$

$$\psi^* = -\frac{\sigma_m^2}{\sigma_r^2}\frac{N\mu + (M-1)\psi}{N\beta}[\lambda_m + \rho_m\sigma_m^2 + \frac{\sigma_m^2}{\sigma_r^2}\frac{1}{N\beta}]^{-1} \tag{22}$$

Proof. The proof is simply algebra once we have established the previous lemmas and plug the conditional variances and expected values into equations (3) and (4).
Equation (3) is equivalent as the following:

$$(\lambda_n + \rho_n\sigma_c^2)X_n = \frac{\sigma_c^2}{\sigma_e^2}i_n + (\frac{\sigma_c^2}{\sigma_s^2}\frac{1}{\lambda_n(N-1)\beta} - 1)p - \frac{\sigma_c^2}{\sigma_s^2}\frac{1}{(N-1)\beta}X_n - \frac{\sigma_c^2}{\sigma_s^2}\frac{(N-1)\mu + M\psi}{(N-1)\beta}$$

solving X_n and match the coefficient gives the solution for μ^*, β^*, γ^* in the theorem.
Similarly, the equation for the uninformed traders can be written as:

$$(\lambda_m + \rho_m\sigma_m^2)Y_m = (\frac{\sigma_m^2}{\sigma_r^2}\frac{1}{\lambda_m N\beta} - 1)p - \frac{\sigma_m^2}{\sigma_r^2}\frac{1}{N\beta}Y_m - \frac{\sigma_m^2}{\sigma_r^2}\frac{(M-1)\psi + N\mu}{N\beta}$$

Solving Y_m from the equation and match the coefficient gives the solution of δ^* and ψ^* in the theorem. □

The previous theorem characterizes the best response function for any informed (uninformed) trader. The next theorem characterizes the SBNE of this game.

Theorem 4.6. *The Symmetric Bayesian Nash equilibrium is characterized as the fixed point of the previous equation systems.*

$$(\mu^*, \beta^*, \gamma^*) = (\mu, \beta, \gamma)$$

$$(\psi^*, \delta^*) = (\psi, \delta)$$

From previous analysis we can conclude that $(\mu^*, \beta^*, \gamma^*)$ and (ψ^*, δ^*) are the best response functions for player n and m respectively while assuming others' strategy parameters are (μ, β, γ) and (ψ, δ) respectively. Therefore once we have a fixed point of this program, it is true that player n or m is optimizing by choosing the same strategy. Since agents are ex-ante identical within groups, we can conclude that for all informed agent in $\{1, 2, 3, ...N\}$ and uniformed agent $\{1, 2, 3, ...M\}$ are optimizing. Therefore no one will have the incentive to deviate given the strategy profile of others. So it is the SBNE of this game.

Notice that $\psi = \mu = 0$ is clearly a solution to the fixed point problem. To simplify the analysis, we could assume $\psi = \mu = 0$ is the equilibrium strategy for all players in the game. This intuition is also supported by the assumption that $\mathbb{E}[\tilde{v}] = 0$. Since the unconditional return of the asset is normalized to 0, the conditional expected value will not consist a constant. Therefore it is sensible to assume other traders will not have a non-zero constant in their demand strategy.

This is equivalent as considering a game where informed traders' strategy is $X_n = \beta i_n - \gamma p$ and $Y_m = -\delta p$. The simplified equation systems is the following:

$$\beta = \frac{\sigma_c^2}{\sigma_e^2}[\lambda_n + \rho_n \sigma_c^2 + \frac{\sigma_c^2}{\sigma_s^2}\frac{1}{(N-1)\beta}]^{-1} \tag{23}$$

$$\gamma = [1 - \frac{\sigma_c^2}{\sigma_s^2}\frac{1}{\lambda_n (N-1)\beta}][\lambda_n + \rho_n \sigma_c^2 + \frac{\sigma_c^2}{\sigma_s^2}\frac{1}{(N-1)\beta}]^{-1} \tag{24}$$

$$\delta = [1 - \frac{\sigma_m^2}{\sigma_r^2}\frac{1}{\lambda_m N\beta}][\lambda_m + \rho_m \sigma_m^2 + \frac{\sigma_m^2}{\sigma_r^2}\frac{1}{N\beta}]^{-1} \tag{25}$$

We could prove the existence of a fixed point to this system using Brouwer fixed point theorem. However, the system of equations are different from Kyle's original solution.[2]

5 Simulation

In this section, I will simulate the equilibrium for various parameter values and show the difference between the proposed solution and Kyle's original solution. The following table shows the simulation result.

[2] Appendix B Equation B.6 and B.7 in the original paper.

σ_z^2	10	20	30	40	50
β	0.0868	0.0903	0.0916	0.0923	0.0927
$\tilde{\beta}$	0.0890	0.0924	0.0936	0.0943	0.0947
γ	0.1554	0.1689	0.1747	0.1779	0.1799
$\tilde{\gamma}$	0.1550	0.1687	0.1745	0.1777	0.1798
δ	0.0702	0.0827	0.0882	0.0914	0.0934
$\tilde{\delta}$	0.0700	0.0825	0.0881	0.0913	0.0933

Table 1: Simulation results
Parameter value
$\sigma_e^2 = \sigma_v^2 = 10; N = M = 5; \rho_m = \rho_n = 0.9$

The proposed solution is denoted as (β, γ, δ) while $(\tilde{\beta}, \tilde{\gamma}, \tilde{\delta})$ is the solution to Kyle (1989). In terms of how traders respond to price, there is a very small difference between Kyle's solution and the proposed solution. However, $\tilde{\beta}$ is always bigger than β by approximately 0.002. This result is consistent with our intuition that imposing symmetry before computing best response seems to provide incentives for agents to respond more to private information as it increases price informativeness.

References

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